Far East Journal of Mathematical Sciences (FJMS)

Published Online: October 2015
http://dx.doi.org/10.17654/FJMSNov2015_769_775
Volume 98, Number 6, 2015, Pages 769-775 **ISSN: 0972-0871**

THE EFFECT OF COMBINING HERBAL AND SYNTHETIC MEDICINES ON STABILITY ANALYSIS OF TUBERCULOSIS SPREADING DISEASE MODEL (CASE STUDY: TUBERCULOSIS IN BOGOR REGION, WEST JAVA, INDONESIA)

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Abstract

Synthetic drugs that commonly used for healing tuberculosis disease are: isoniazid, rifampicin, ethambutol and pyrazinamide. Healing tuberculosis disease takes a very long time usually from 6 until 9 months. Besides that, patients also need to pay a lot of money for the treatments. Moreover, tuberculosis can lead to death if the patients did not aware of the disease. Therefore, the spreading of tuberculosis disease needs to be controlled. Nowadays, many people tend to have natural medicine (herbal medicine) because it is believed that taking additional herbal medicine can decrease the recovery time and also can increase immune system of the body. In this paper, we present the

Received: July 7, 2015; Accepted: August 11, 2015
2010 Mathematics Subject Classification: 92-XX.
Keywords and phrases: tuberculosis, SIR model, herbal, fixed point stability.
Communicated by K. K. Azad

effect of combining herbal and synthetic medicines in the SIR model of tuberculosis disease. The idea is to add an additional parameter corresponding to the herbal medicine onto the underlying SIR (susceptible, infected, recovered) model for the tuberculosis disease. After the model has been constructed, we analyze the model analytically by defining the compartments of the model, the system of differential equations of the model, some assumption to the model. After the model has been defined completely, we search for fixed points, eigenvalues and eigenvector for the model. Finally, we analyze the fixed points with respect to Routh-Hurwitz criterion, then we compute the basic reproduction number to give us the information about the spreading level of tuberculosis disease. The output of this research is an SIR model of tuberculosis spreading disease with additional parameter, namely, herbal medicine. We hope this model can be used to predict and also to control the tuberculosis disease in the society. Furthermore, we can prevent tuberculosis as an endemic disease in the future.

1. Introduction

Curing tuberculosis disease takes a very long time and a lot of money. Synthetic drugs commonly used for healing tuberculosis are: isoniazid, rifampicin, ethambutol and pyrazinamide. Some people use alternative medicine used as a supporting medicine in healing tuberculosis. They use herbal medicine to increase the stamina of the body and to maintain the healthiness. Moreover, since the herbal medicine is more ‘natural’, many people use it simultaneously with the synthetic medicine. This research is focusing on understanding the use of the herbal medicine in affecting tuberculosis treatment. The idea is to add a herbal parameter to the SIR model (susceptible, infected, recovered). After the model is fixed, we analyze the model and try to use the model to predict and control the spreading of the disease by changing the parameters. Using this result, we hope it can be used as a control management so we can prevent tuberculosis from being endemic disease in the future. This research is a follow up research from [3], referring to SIR model in [5].

2. Tuberculosis Spreading Model with Herbal Medicine

The population in the tuberculosis spreading model is divided into three parts, i.e., groups of susceptible individuals $(s(t))$ based on assumption stated in [1], with concerning the condition in Indonesia, groups of individuals who are infected with TB disease $(I(t))$ and group of individuals which have been recovered $(R(t))$. The data in this model acquired from a hospital in Bogor regency, West Java, Indonesia.

The compartment model of the spreading of tuberculosis is as follows:

Referring to Figure 1, groups of susceptible individuals $(s(t))$ will increase steadily based on π (the birth) and decrease because the death (μ) uses of herbal medicine and direct contact with an infected individual group β. Groups of individuals who are infected with TB disease $(I(t))$ will increase with rate β, but will also decreased due to natural mortality with rate μ, mortality due to TB with rate (μ_t) and recovery with the rate γ. Group of individuals who are recovered $(R(t))$ will decrease because of natural mortality and will increase with rate γ.

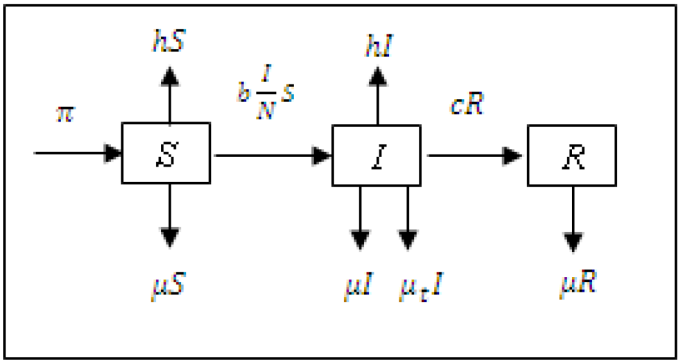

Figure 1. Compartment model of TB disease spreading with herbal parameter.

Based on the assumptions and the compartments of the model, the mathematical model was obtained in the form of differential equations as follows:

$$\frac{dS}{dt} = \pi - b\frac{I}{N}S - \mu S - hS,$$

$$\frac{dI}{dt} = b\frac{I}{N}S - (\mu + \mu_t + c + h)I,$$

$$\frac{dR}{dt} = cI - \mu R \tag{1}$$

with $N = S + I + R$.

3. Analysis of Tuberculosis Disease Spreading Model with Herbal Parameter

The fixed point of equations (1) was obtained when individual groups growth reached zero or $\frac{dS}{dt} = 0$, $\frac{dI}{dt} = 0$, $\frac{dR}{dt} = 0$. The fixed points are obtained through the following phases:

(1) If $\frac{dI}{dt} = 0$, then we obtain $I_0^* = 0$. Substituting this value to $\frac{dR}{dt} = 0$, we obtain $R_0^* = 0$. Substituting both values to $\frac{dS}{dt} = 0$, we get $S_0^* = \frac{\pi}{h + \mu}$. Hence, the first fixed point is $T_1(S_0^*, I_0^*, R_0^*) = \left(\frac{\pi}{h + \mu}, 0, 0\right)$.

(2) The second fixed point is obtained when $\frac{dS}{dt} = 0$ and $\frac{dI}{dt} = 0$. With the same method as before, we obtained the second fixed point as follows: $T_2(S_1^*, I_1^*, R_1^*) = \left(\frac{\pi P}{Q}, \frac{\pi\mu}{UQ}, \frac{c\pi Z}{UQ}\right)$, where $P = \mu + c$, $Q = ch + b\mu$, $Z = b - c - h - \mu - \mu_t$ and $U = c + h + \mu + \mu_t$.

The rate of disease spreading can be expressed using basic reproduction ratio (R_0). It is obtained by differentiating the growth rate of the infected population with respect to I. Hence, $R_0 = \frac{b}{c + h + \mu + \mu_t}$.

Now, we analyze the stability of the fixed point using Routh-Hurwitz criterion [6] in the disease free situation $(R_0 < 1)$.

(1) Stability of first fixed point

First, we compute a_1 and a_2, where

$$a_1 = Tr(M) = -(m_1 1 + m_2 2 + m_3 3) = c + 2h + 3\mu - b,$$

$$a_2 = h\mu + \mu^2 + (h + 2\mu)(-b + c + h + \mu + \mu_t),$$

$$a_3 = (h\mu + \mu^2)(-b + c + h + \mu + \mu_t).$$

The fixed point is stable if $a_1 > 0$, $a_3 > 0$ and $a_1 a_2 - a_3 > 0$. The condition is satisfied if $(-b + c + h + \mu + \mu_t) > 0$. Hence, we conclude that the first fixed point is stable under the condition $R_0 < 1$ (disease free equilibrium).

(2) Stability of second fixed point

With the same method for analyzing the first fixed point, we have $a_1 > 0$, $a_3 < 0$ and $a_1 a_2 - a_3 > 0$. If $Z < 0$, then $Z = b - c - h - \mu - \mu_t$ and if $U > 0$, then $U = c + h + \mu + \mu_t$. This fact contradicts the Routh-Hurwitz criterion, hence we can say that the second fixed point

$$T_2(S_1^*, I_1^*, R_1^*) = \left(\frac{\pi P}{Q}, \frac{\pi\mu}{UQ}, \frac{c\pi Z}{UQ}\right)$$

is *unstable* under the condition $R_0 < 1$.

The spread of tuberculosis disease in Bogor regency, West Java, Indonesia with $\pi = 4.8$, $b = 0.005$, $h = 0.001$, $\mu = 0.009$ and $\mu_t = 0.00000001$ was obtained as follows:

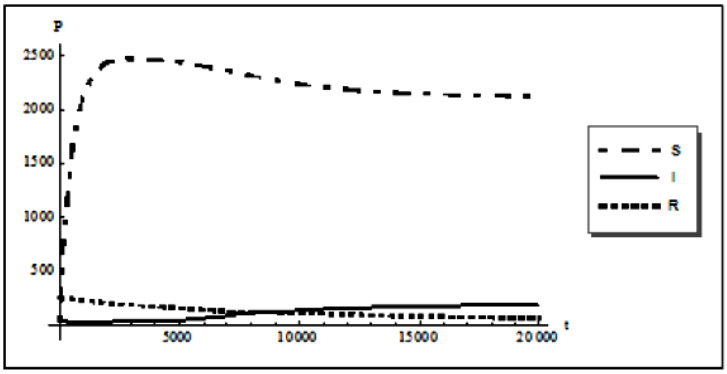

Figure 2. Spreading of tuberculosis in Bogor region with respect to herbal medicine.

Figure 2 shows that the population of patients with tuberculosis in Bogor, West Java changed over time. In the first 5000 days, tuberculosis-infected population has decreased, hence the population of susceptible and recovered people has increased. This means that the spreading of tuberculosis can be controlled (within this period). But, we can see that the condition has changed afterwards. This situation shows that the herbal medicine acts as a supporting medicine for the synthetic medicine, because the herbal medicine can increase the body immune system so that the synthetic medicine can work more effectively in curing tuberculosis disease.

4. Conclusion

Tuberculosis spreading model with herbal medicine has two fixed points. The first fixed point acts as a disease free fixed point, while the second fixed point is an endemic fixed point. We also have that the basic reproduction value is $R_0 = \frac{b}{c + h + \mu + \mu_t}$. In the treatment of tuberculosis, herbal medicine can support the synthetic medicine so that the patient can heal more quickly. Since, in practice, herbal medicine can increase the immune system of our body. With this result, we can control the spreading of tuberculosis disease in the future.

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