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[3] D.O. Hebb, The Organization of Behavior, Wiley, New York, 1949.

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Stability Analysis Model of Spreading and Controlling of Tuberculosis

(Case Study: Tuberculosis in Bogor Region, West Java, Indonesia)

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Abstract

This research described the phenomenon of the spreading of tuberculosis in Bogor West Java Indonesia into a mathematical model used SIR model (Susceptible, Infected, Recovered model), because of tuberculosis is an infectious disease that made death, needed a treatment time of 6 to 9 months and the cost was expensive, so that the rate of spreading of tuberculosis must be controlled. Analytical level consisted of fixed point searching, analyzing the stability of a fixed point, determining the basic reproduction ratio, then analyzed numerically used Mathematica software. The results of this research showed that the most influential parameters in the spreading of tuberculosis, so that the spreading of tuberculosis in Bogor can be solved. Hence, it would prevent poverty and unproductiveness cases, and gave government policy which was related to control of spreading tuberculosis.

Mathematics Subject Classification: 93C10

Keywords: tuberculosis, SIR models, stability of a fixed point, basic reproduction ratio

1 Introduction

Tuberculosis (TB) is one of epidemic and infectious disease which caused by the Mycobacterium tuberculosis bacterium that not only attacks the lungs but can attack other organs, the brain, kidney, intestine, bone and skin [6]. Mathematical modeling is one of ways to solve this problem. Mathematical modeling can assist in predicting and controlling the spreading of disease through estimating the model parameters that influenced the spreading of the disease. The result can be used as a recommendation in determining strategies for controlling TB, so it will not be an epidemic case.

Based on the literature, it is informed that the study of tuberculosis (TB) in mathematics are rarely performed. In general, many studies conducted analyzing about the classical model for TB disease and find out equilibrium point. Several studies have been done by [7] which uses dynamic analysis to analyze sensitivity of TB with no treatment, [1] added a simple vaccine for tuberculosis SIS model and indicate that the vaccine can reduce the basic reproduction number for TB disease [9] analyze tuberculosis with exogenous reinfection, [4] analyzed of spreading TB with Runge-Kutta 4 method which reveals that the spread of TB can be controlled from the activities of the epidemic by reducing the transmission rate and increase the rate of health, meanwhile [2] made and analyzed the transmission dynamics of tuberculosis model.

2 Fixed Point

Suppose given a system of differential equations (DE) as follows

$$\frac{dx}{dt} = \dot{x} = f(x), x \in \mathbb{R}^n. \tag{1}$$

A point x^* that satisfied $f(x^*) = 0$ is called equilibrium point or fixed point of the equation system by [8].

3 Routh-Hurwitz Criterion

Suppose $a_1, a_2, ..., a_k$ are real numbers, $a_j = 0$ if $j < k$, then the eigenvalues of the characteristic equation $p(\lambda) = \lambda^k + a_1\lambda^{(k-1)} + a_1\lambda^{(k-2)} + \cdots + a_k = 0$ have negative real parts if the determinant of the matrix H_j are positive. Furthermore the Hurwitz matrix H_j is:

$$\begin{pmatrix} a_1 & 1 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & 1 & \cdots & 0 \\ a_5 & a_4 & a_3 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{2j-1} & a_{2j-2} & a_{2j-3} & a_{2j-4} & \cdots & a_j \end{pmatrix}$$

with $H_j = (h_{lm})$ and

$$h_{lm} = \begin{cases} a_{2l-m}, & ; 0 < 2l - m < k \\ 1, & ; 2l = m \\ 0, & ; 2l < m \text{ or } 2l > k + m. \end{cases}$$

The eigenvalues of the characteristic equation have negative real parts (fixed point $\dot{x}$ was stable) if and only if the determinant of every positive Hurwitz matrix, that is $H_j > 0$ for $j = 1, 2, ..., k$, so that based the Routh-Hurwitz criteria for each $k, k = 2, 3, 4$ that mentioned that the fixed point $\dot{x}$ was stable if and only if (for $k = 2, 3, 4$),

1. $k = 2,\ a_1 > 0, a_2 > 0$
2. $k = 3,\ a_1 > 0, a_3 > 0, a_1a_2 > a_3$

by [5]

4 Basic Reproduction Ratio

Basic reproduction ratio is the average of susceptible individuals infected directly by other individuals who was infected if an infected individual was belonging into the whole population whose still vulnerable.

1. If $R_0 < 1$, then the disease will disappear.
2. If $R_0 = 1$, then the disease will be settled.
3. If $R_0 > 1$, then the disease will be endemic.

by [3]

5 SIR Model for Spread of Tuberculosis

SIR models for the spreading of TB disease population was divided into three groups, groups of susceptible individuals ($s(t)$) which will increase steadily based on π (the birth) and decrease because the death (μ) and direct contact with an infected individual groups β , groups of individuals who are infected with TB disease ($I(t)$), which will increase with rate β, decreased due to natural mortality μ and mortality due to TB (μ_t) and have been recovery with the rate γ, group of individuals who recover ($R(t)$) who decreased because of natural mortality and increased because of there was the recovery with rate γ. The compartment model of the spreading of tuberculosis as follows:

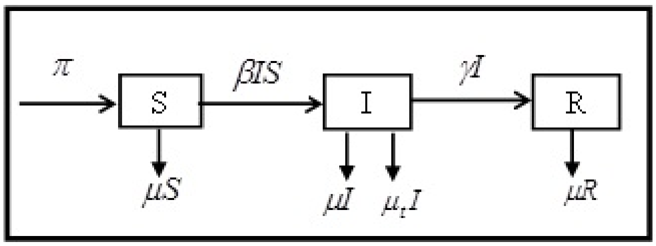

Figure 1: Compartment Model of TB Disease Spreading

Based on the assumptions and the compartments of the model, the mathematical model was obtained in the form of differential equations as follows:

$$\begin{aligned} \frac{dS}{dt} &= \pi - \beta SI - \mu S \\ \frac{dI}{dt} &= \beta SI - (\mu + \mu_t + \gamma)I \\ \frac{dR}{dt} &= \gamma I - \mu R \end{aligned} \tag{2}$$

with $N = S + I + R$

6 Analysis of Tuberculosis Disease Spreading Model

6.1 Fixed Point

In the model of the spreading analysis model, the first step was to determined fixed points of the model which was formed. The fixed point of Equations 2 was obtained when individual groups growth reached zero or $\frac{dS}{dt} = 0$, $\frac{dI}{dt} = 0$, $\frac{dS}{dt} = 0$, but because $\frac{dR}{dt} = 0$ did not appear in the other equations, the fixed point obtained at the time $\frac{dS}{dt} = 0$ and $\frac{dI}{dt} = 0$. The fixed point which is obtained through the following phases:

1. If $\frac{dS}{dt} = 0$ and $\frac{dI}{dt} = 0$, then we obtain I_0^* and $S_1^* = \frac{\mu+\mu_t+\gamma}{\beta}$, then if I_0^* is substituted into $\frac{dS}{dt} = 0$, furthermore we obtain $S_0^* = \frac{\pi}{\mu}$ so that the first fixed point is $T_1(S_0^*, I_0^*) = (\frac{\pi}{\mu}, 0)$, this was disease free equilibrium.

2. If $S_1^* = \frac{\mu+\mu_t+\gamma}{\beta}$ substituted into $\frac{dS}{dt} = 0$, then we obtain value $I_1^* = \frac{1}{\beta}(\frac{\pi}{S_1^*} - \mu) = (\frac{\pi}{\mu+\mu_t+\gamma} - \frac{\mu}{\beta})$, so that the second fixed point is $T_2(S_1^*, I_1^*) = (\frac{\mu+\mu_t+\gamma}{\beta}, (\frac{\pi}{\mu+\mu_t+\gamma} - \frac{\mu}{\beta}))$, that showed epidemic has been occurred, but if it should be disease-free then $(\frac{\pi}{S_1^*} - \mu) \leq 0$ or $S_1^* \geq \frac{\pi}{\mu}$, in other word, the parameters should satisfy $\frac{\mu+\mu_t+\gamma}{\beta} \geq \frac{\pi}{\mu}$.

6.2 Jacobian Matrix

In this step, determined the Jacobian matrix of equation 2 as follows:

$$M_J = \begin{pmatrix} \frac{\partial S}{\partial S} & \frac{\partial S}{\partial I} \\ \frac{\partial I}{\partial S} & \frac{\partial I}{\partial I} \end{pmatrix} = \begin{pmatrix} -\beta I - \mu & -\beta S \\ \beta I & \beta S - \mu - \mu_t - \gamma \end{pmatrix},$$

after obtained the Jacobian matrix, then the stability of fixed point will be analyzed.

6.3 Stability Analysis of Fixed Point

In this step, both of the points will be analyzed about fixed point stability used the Routh-Hurwitz criteria as follows:

1. $T_1(S_0^*, I_0^*) = (\frac{\pi}{\mu}, 0)$ is substituted to M_J so that we obtain $M_J = \begin{pmatrix} -\mu & -\beta\frac{\pi}{\mu} \\ 0 & \beta\frac{\pi}{\mu} - \mu - \mu_t - \gamma \end{pmatrix}$, then $a_1 = 2\mu - \beta\frac{\pi}{\mu} + \mu_t + \gamma$ or $a_1 > 0$ and $a_2 = \mu^2 + \mu\mu_t + \gamma\mu - \beta\mu$ or $a_2 > 0$. Based on the Routh-Hurwitz criteria, the conditions was satisfied, so the point $T_1(S_0^*, I_0^*) = (\frac{\pi}{\mu}, 0)$ said to be stable, which means that the disease free equilibrium fixed point or did not happened epidemic.

2. $T_2(S_1^*, I_1^*) = (\frac{\mu+\mu_t+\gamma}{\beta}, (\frac{\pi}{\mu+\mu_t+\gamma} - \frac{\mu}{\beta}))$ is substituted into the Jacobian matrix, we obtain $M_J = \begin{pmatrix} -\frac{\beta\pi}{\pi+\pi_t+\gamma} & -\mu - \mu_t - \lambda \\ \frac{\beta\pi}{\mu+\mu_t+\gamma} - \mu & 0 \end{pmatrix}$, then $a_1 = \frac{\beta\pi}{\mu+\mu_t+\gamma}$ or $a_1 > 0$ and $a_2 = \beta\pi - \mu(\mu + \mu_t + \gamma)$ or $a_2 < 0$. Because $a_1 > 0$ and $a_2 < 0$, that was contradictory with the conditions which was satisfied in the Routh-Hurwitz criterion, then it can be concluded both of fixed point is unstable.

6.4 Basic Reproduction Ratio R_0

If we want to achieve disease-free or free of infection, it should be $(\frac{\pi}{S_1^*}) \leq 0$ or $S_1^* \geq \frac{\pi}{\mu}$, in other words $\frac{\mu+\mu_t+\gamma}{\beta} \geq \frac{\pi}{\mu}$ was threshold or thresholds epidemic, so that it can be concluded:

1. If $\frac{\mu+\mu_t+\gamma}{\beta} < \frac{\pi}{\mu}$, then there was epidemic or a group which was infected rise.

2. If $\frac{\mu+\mu_t+\gamma}{\beta} \geq \frac{\pi}{\mu}$, then epidemic will not occur or groups of individuals which was infected reaches zero or there is not a group which infected individuals.

The basic reproduction can be obtained through the following steps as follows: Due to the free of disease or $S_1^* \geq \frac{\pi}{\mu}$ or $\frac{\pi}{\mu S_1^*} \leq 1$, the same meaning with $\frac{\pi}{\mu(\frac{\mu+\mu_t+\gamma}{\beta})} \leq 1$ or $\frac{\beta\pi}{\mu(\mu+\mu_t+\gamma)} - 1 \leq 0$, in order to obtained the basic reproduction ratio was $R_0 = \frac{\beta\pi}{\mu(\mu+\mu_t+\gamma)}$. After R_0 obtained, it can be used to measure the rate of spreading of a disease that was, if every patient could only transmit the disease to a vulnerable person or a new patient and eventually the disease will disappear then $R_0 < 1$, this means that epidemic will not happened, whereas if every patient could infected in more of a new patient and eventually the disease will plague then $R_0 > 1$, this means that epidemic is happened.

7 Numerical Analysis Model for Disease Spreading of Tuberculosis

This stage described about the pattern of the spreading of tuberculosis based on tuberculosis patients data in Bogor West Java within 2010 until 2012 with $\pi = 4.8, \beta = 0.005, \gamma = 0.027, \mu = 0.009, \mu_t = 0.00000001$ was obtained as follows:

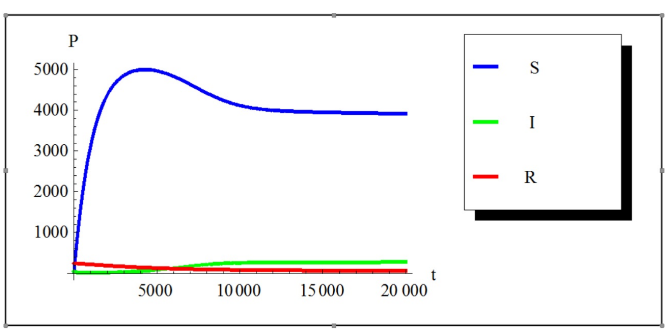

Figure 2: Spreading of Tuberculosis in Bogor Region

Figure 2 showed that the population of patients with tuberculosis in Bogor, West Java changed over time, within 5000 days for tuberculosis-infected population has decreased, this made population susceptible has increased and recover populations also increased, then showed that the condition spreading TB can be controlled. But if treatment did not given intently and if they did not healthy lifestyle and did not the other preventive action then the next time the population whose infected with TB will be increasing, it will make the disease cannot be controlled or become epidemic.

8 Conclusion

In controlling spreading of tuberculosis is necessary to measure of spreading disease rate R_0, it should be $R_0 < 1$ to prevent the epidemic. In this case in order to satisfy the following condition: $R_0 = \frac{\beta\pi}{\pi(\pi+\pi_t+\gamma)} < 1$, it is required that $\beta < \mu(\mu + \mu_t + \gamma)$. In other words that that the infection rate should be smaller, because the natural mortality rate, or mortality rate which was caused by tuberculosis cannot be increased. The requirement can be satisfied by decreasing infection rate and increased the recovery rate through intensive treatment, so that it can be concluded that the parameters which was influenced in tuberculosis spreading model were β andγ parameters.

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